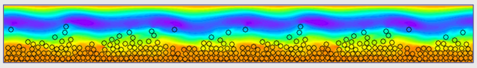
Direct numerical simulation of incipient sediment motion and hydraulic conveying



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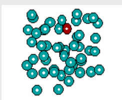
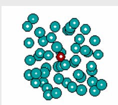
"New Frontiers in Chemical & Biochemical Engineering"
Symposium in honor of Professors *Anastasios Karabelas* and *Stavros Nychas*
Thessaloniki, November, 2009

Motivation

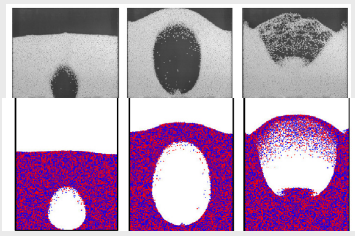
Particle-laden flows are at the heart of several chemical engineering processes, i.e.

Sedimentation

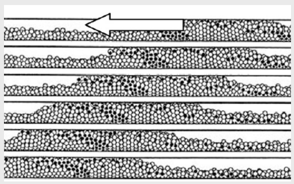




Fluidization



Pneumatic conveying



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Motivation

Particle-laden flows are ubiquitous in nature

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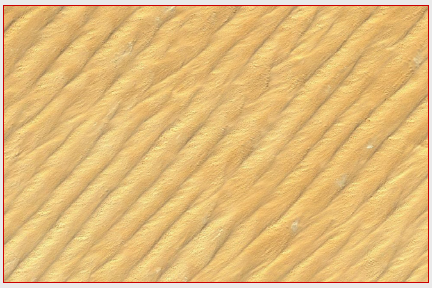
Motivation

Particle-laden flows are ubiquitous in nature



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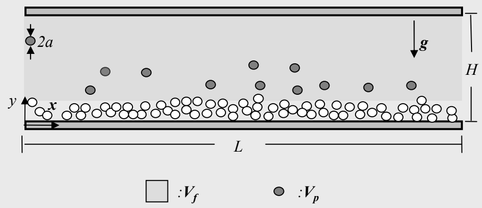
Problem definition



The motion of fluid and particles is studied in a plane channel under the action of a constant pressure gradient

Governing equations

Navier-Stokes and continuity equations for the fluid

$$\rho(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}.\nabla \boldsymbol{u}) = -\nabla P + \mu \nabla^2 \boldsymbol{u} + \rho \boldsymbol{g}$$

$$\nabla.\boldsymbol{u} = 0$$

Conservation of linear and angular momentum for the particles

$$m\frac{d\boldsymbol{U}}{dt} = m\boldsymbol{g} + \boldsymbol{F}_{NH} + \boldsymbol{F}_H$$

$$I\frac{d\boldsymbol{\omega}}{dt} = \boldsymbol{T}_{NH} + \boldsymbol{T}_H$$

Hydrodynamic forces and torques

$$\boldsymbol{F}_H = \oint_S \boldsymbol{\sigma}.\boldsymbol{n} dS$$

$$\boldsymbol{T}_H = \oint_s \boldsymbol{r} \times \boldsymbol{\sigma}.\boldsymbol{n} dS$$

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Dimensionless parameters

$Re = \rho U_0 H / \mu$ **Reynolds number**

$Ar = \dfrac{\rho \Delta\rho g H^3}{\mu^2}$ **Archimedes number (= 10^6)**

$\hat{\rho} = \rho_p / \rho_f$ **Density ratio (= 2)**

$\hat{a} = a / H$ **Particle size (= 0.04)**

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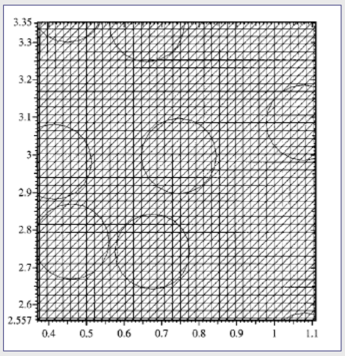
Numerical method

Distributed Lagrange Multiplier/Fictitious Domain Method of Glowinski et al. (1999)



A fixed grid is used!

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Numerical method

$$
\begin{aligned}
&\rho_f \int_\Omega \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right]\cdot \mathbf{v}\,d\mathbf{x} - \int_\Omega p\nabla\cdot\mathbf{v}\,d\mathbf{x} + 2\nu\int_\Omega \mathbf{D}(\mathbf{u}):\mathbf{D}(\mathbf{v})\,d\mathbf{x}\\
&- \sum_{j=1}^{J}\langle \lambda_j, \quad \mathbf{v} - \mathbf{Y}_j - \theta_j \times \overrightarrow{\mathbf{G}_j\mathbf{x}}\,\rangle_j + \sum_{j=1}^{J}(1-\rho_f/\rho_j)M_j\frac{d\mathbf{V}_j}{dt}\cdot\mathbf{Y}_j\\
&+ \sum_{j=1}^{J}(1-\rho_f/\rho_j)\left(\mathbf{I}_j\frac{d\omega_j}{dt} + \omega_j\times\mathbf{I}_j\omega_j\right)\cdot\theta_j = \rho_f\int_\Omega \mathbf{g}\cdot\mathbf{v}\,d\mathbf{x}\\
&+ \sum_{j=1}^{J}(1-\rho_f/\rho_j)M_j\mathbf{g}\cdot\mathbf{Y}_j, \qquad \forall\mathbf{v}\in\left(H_0^1(\Omega)\right)^d, \forall\mathbf{Y}_j\in\mathbb{R}^d, \forall\theta_j\in\mathbb{R}^3,
\end{aligned}
$$

$$\int_\Omega q\nabla\cdot\mathbf{u}\,d\mathbf{x} = 0, \qquad \forall q\in L^2(\Omega),$$

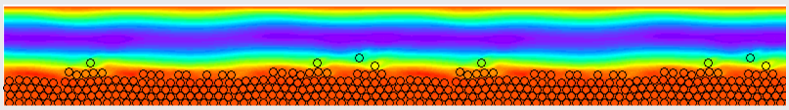
$$\langle \mu_j, \mathbf{u}(t) - \mathbf{V}_j(t) - \omega_j(t)\times\overrightarrow{\mathbf{G}_j(t)\mathbf{x}}\rangle_j = 0, \qquad \forall\mu_j\in\Lambda_j(t), \forall j = 1,\ldots,J,$$

$$\frac{d\mathbf{G}_j}{dt} = \mathbf{V}_j, \qquad \forall j = 1,\ldots,J,$$

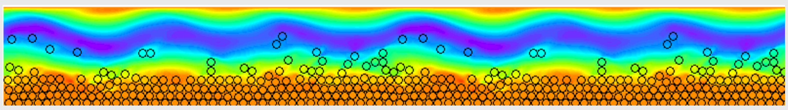
Variational formulation of the governing equations

Flow and particle behavior at increasing *Reynolds* numbers

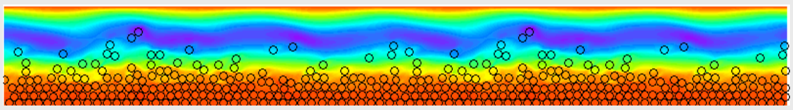
Re = 3500

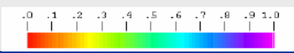


Re = 6000



Re = 8000





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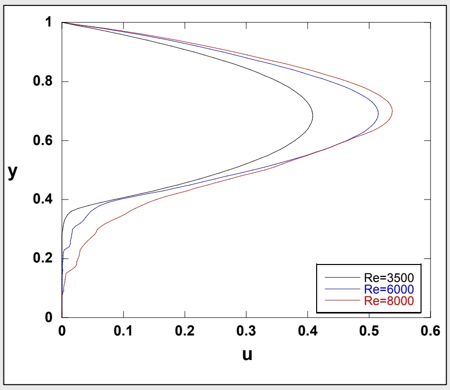
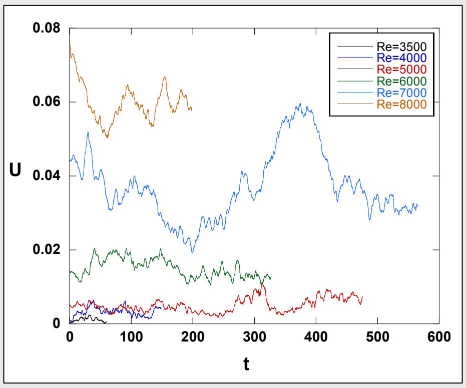
Velocity as a function of *Reynolds* number





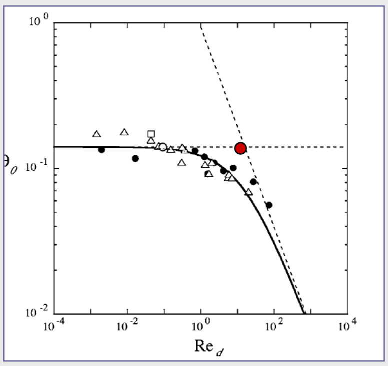
Time-averaged fluid velocity

Average particle velocity

Critical *Shields* number

The onset of particle motion corresponds to a critical *Shields* number of 0.14

$$\theta = \frac{\tau_s}{2\Delta\rho g a}$$



(*Loiseleux et al, 2005*)

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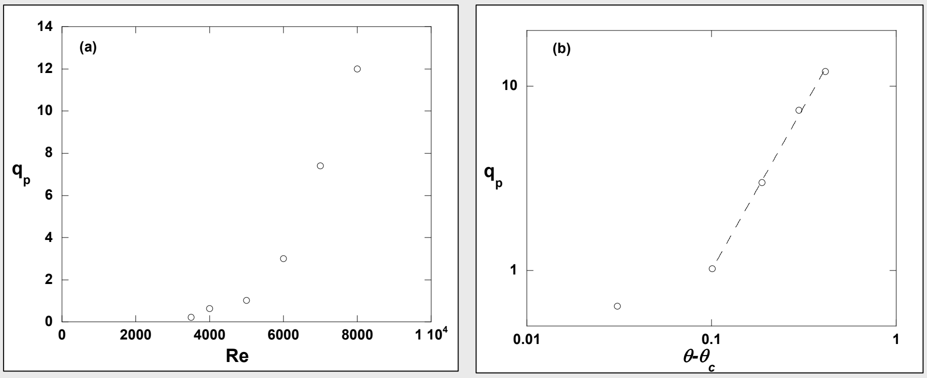
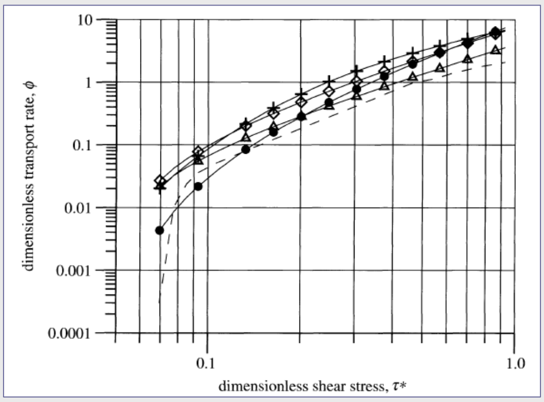
Particle flux



Particle flux vs. *Re* or vs. excess *Shields* number

Particle flux



(*Heald et al, 2004*)

Various power-law expressions have been proposed
The exponents usually range between 1.5 – 2.0

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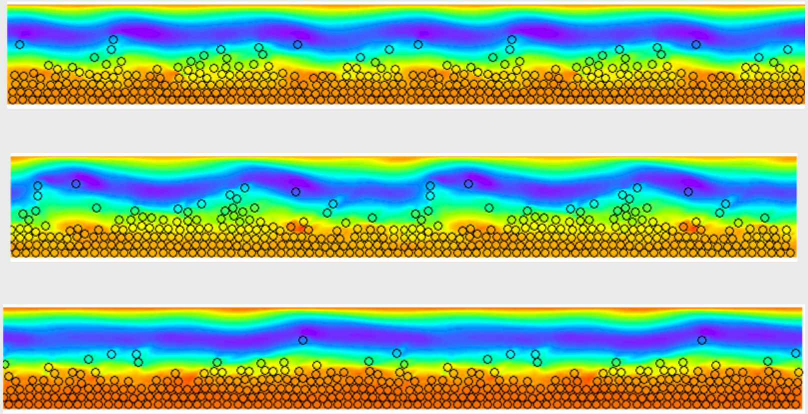
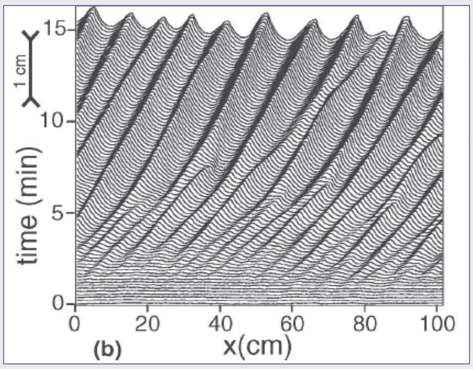
Flow instabilities



Horizontal velocity contours at various time instants for *Re* = 8000

Flow instabilities



Sand ripple coarsening is usually observed in experiments (*i.e. Andreotti et al, 2006*)

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Conclusions

- *A DNS methodology (DLM/FDM) was implemented, coupled with a m-DEM model for particle collisions*

- *Encouraging results were found for the critical Shields number and the particle flux*

- *A mechanism of hydrodynamic instability in the flowing sheared particle layer appears to be operative*